Asymptotic quasinormal modes of a noncommutative geometry inspired Schwarzschild black hole

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Abstract

We study the asymptotic quasi-normal modes for the scalar perturbation of the non-commutative geometry inspired Schwarzschild black hole in (3+1) dimensions. We have considered $M \geq M_0$, which effectively correspond to a single horizon Schwarzschild black hole with correction due to non-commutativity. We have shown that for this situation the real part of the asymptotic quasi-normal frequency is proportional to ln(3). The effect of non-commutativity of spacetime on quasi-normal frequency arises through the constant of proportionality, which is Hawking temperature $T_H(\theta)$. We also consider the two horizon case and show that in this case also the real part of the asymptotic quasi-normal frequency is proportional to ln(3).

Keywords: Non-commutative Schwarzschild black hole, Asymptotic quasi-normal modes.

1 Introduction

Black hole spacetime, when perturbed by external fields, the perturbations are radiated away and the black hole spacetime returns to its equilibrium. The radiated waves, called quasi-normal modes [1, 2, 3] are characterized by complex frequency ω. They depend only on the parameters of the black hole spacetime and are independent of the details concerning the initial perturbation. On the other hand asymptotic quasi-normal modes (quasi-normal modes which damp infinitely fast) do not radiate at all and therefore they can be interpreted as the fundamental oscillation for the black hole spacetime.

Asymptotic quasi-normal modes are supposed to have a role to play in the quest for a theory of quantum gravity [4, 5]. It is further suggested [5] that asymptotic quasi-normal frequency may help to fix certain parameters in loop quantum gravity. The key factor which makes people believe that asymptotic quasi-normal modes have important role in quantum gravity is the real part of the asymptotic quasi-normal frequency, which was supposed to have the form $Re(\omega) \sim \ln k$ (k is the natural number). But this fact is seen to be true for d dimensional ($d > 3$) Schwarzschild black holes [6, 7], not for all black hole spacetimes. For example, for four dimensional Reissner-Nordström black hole, the asymptotic quasi-normal frequencies have complicated form [7], which is not of the desired from $Re(\omega) \sim \ln k$. For four dimensional Schwarzschild de Sitter (dS) and Schwarzschild Anti-de Sitter (AdS) black hole, again the real part of the asymptotic quasi-normal frequency [8] does not have the form $Re(\omega) \sim \ln k$. So, the important question of universality of the real part of the asymptotic quasi-normal frequency of black hole as suggested in [4, 5] is evidently not valid [9, 10, 11]. Schwarzschild spacetime is in active investigation [12, 13, 14] for many years.

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But all the studies have been performed for the black holes, where spacetime are commutative. Non-commutative spacetime, on the other hand gets special interest due to the prediction of string theory [15, 16, 17, 18, 19], which along with the brane-world scenario [20], lead to the fact that spacetime could be non-commutative. One more important thing about studying non-commutative geometry is probably the various kind of divergence, which appear in General Relativity. It is believed that spacetime non-commutativity may cure this divergence in General Relativity. The analysis of (1+1) and (3+1) dimensional linearized General Relativity [21, 22] reinforce this belief. Schwarzschild black hole has been studied taking non-commutative space into account [23, 24, 25] and it has been shown that to the first order of the non-commutative parameter θ there is no effect in the spacetime metric of the black hole, but in second order there is a effect of non-commutative space in the metric if the linear momentum in the plane perpendicular to the non-commutative parameter is not zero. On the other hand Schwarzschild black hole in non-commutative spacetime has been studied in [26, 27]. It has been shown that the modified metric due to non-commutativity of spacetime does not allow the black hole to decay beyond a critical mass $M_0 = 1.9\sqrt{\theta}$ through Hawking radiation and Hawking temperature does not diverge at all, rather it reaches a maximum value before cooling down to absolute zero.

Having said so much about the non-commutative black hole, it naturally comes to mind that, how does non-commutativity affect the quasi-normal modes, especially to the real part of the asymptotic quasi-normal frequency? It's an open question, and so far has not been answered. In our present work, we are going to address this issue in detail. We will calculate the asymptotic quasi-normal modes for the non-commutative geometry inspired Schwarzschild black hole [26], where non-commutativity can be taken as the correction to the Schwarzschild black hole metric and goes to zero when the strength of noncommmutativity goes to zero. We will show that although the geometry is modified in presence of non-commutativity, the real part of the asymptotic quasi-normal frequency remains proportional to ln(3).

We have organized the paper as follows: In Sec. (2), we discuss the non-commutative geometry inspired Schwarzschild black hole [26]. In Sec. (3), we find out analytically the asymptotic quasi-normal modes and calculate the asymptotic quasi-normal frequency of the single horizon black hole spacetime. In Sec. (4), we find out analytically the asymptotic quasi-normal modes and calculate the asymptotic quasi-normal frequency of the two horizon black hole spacetime. In Sec. (5), we conclude with some discussion.

2 Non-commutative geometry inspired Schwarzschild black hole

The Schwarzschild black hole is usually modified when non-commutative spacetime is taken into account and the modified metric is given by (in units of $G_N = 1, c = 1$) [26]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\phi^2\right), \tag{2.1}$$

with

$$f(r) = 1 - \frac{4M}{r\sqrt{\pi}}\,\gamma(3/2\,, r^2/4\theta\,) \tag{2.2}$$

where r, ϑ, ϕ are the spherical coordinates, θ is the strength of non-commutativity of spacetime and M is the mass of the black hole. The upper incomplete Gamma function $\gamma\left(3/2\,, r^2/4\theta\,\right)$ is given by:

$$\gamma\left(3/2\,, r^2/4\theta\,\right) \equiv \int_0^{r^2/4\theta} dt\, t^{1/2} e^{-t}\,. \tag{2.3}$$

The horizons can be found from the equation,

$$f(r) = 1 - \frac{4M}{r\sqrt{\pi}}\,\gamma(3/2\,, r^2/4\theta\,) = 0\,. \tag{2.4}$$

But it is impossible to solve this equation analytically in exact form. However, it is possible to solve it numerically [26] and get the radius of the horizons. Numerical solution shows that it has different horizons for different values of the mass M of the black hole:

1. For $M > M_0 = 1.9 \times (\sqrt{\theta})/G$, there is two distinct horizons;
2. For $M = M_0 = 1.9 \times (\sqrt{\theta})/G$, there is one degenerate horizon at $r_0 = 3.0 \times \sqrt{\theta}$, which corresponds to extremal black hole;
3. For $M < M_0 = 1.9 \times (\sqrt{\theta})/G$, there is no horizon.

Our present interest is for the case of two horizons, because by increasing the mass M of the black hole, i.e. for $M \gg M_0$, the inner horizon can be shrunk to zero and the outer horizon then approaches to Schwarzschild value $r_h = 2M$. So effectively we can consider it as a single horizon Schwarzschild black hole, where the event horizon is modified due to non-commutativity. The event horizon for this case can be written in terms of upper incomplete gamma function

$$r_h \approx 2M \left[1 - \frac{2}{\sqrt{\pi}} \gamma\left(3/2\,, M^2/\theta\right) \right], \tag{2.5}$$

and the Hawking temperature for the black hole is given by

$$\begin{aligned} T_h(\theta) &\equiv \frac{1}{4\pi}\left[\frac{df(r)}{dr}\right]_{r=r_h} \\ &= \frac{1}{4\pi\, r_h}\left[1 - \frac{r_h^3}{4\,\theta^{3/2}}\,\frac{e^{-r_h^2/4\theta}}{\gamma\,(3/2\,, r_h^2/4\theta)}\right]. \end{aligned} \tag{2.6}$$

It is easy to see that for $\theta \to 0$ Eq. (2.5) and Eq. (2.6) reduces to the pure Schwarzschild value.

3 Asymptotic quasi-normal mode analysis for the single horizon black hole

In this section we obtain asymptotic quasi-normal frequency corresponding to the scalar perturbations for spherically symmetric (3+1) dimensional non-commutative geometry inspired Schwarzschild black-hole [26] discussed in previous section. Before proceeding further one important point we would like to mention is that, in [26], the effect of non-commutativity has been introduced in the right hand side of the Einstein equation as modified matter distribution keeping the Einstein tensor in the left hand side intact. In this same spirit we also consider the Klein Gordon equation on curved spacetime, where the metric is taken from Ref. [26] and we have kept everything except the metric like commutative spacetime. Once we assume this, then the evolution equation for the scalar field perturbation follows directly from the massless, minimally coupled scalar field propagating in the line element Eq. (2.1). The scalar perturbation is governed by,

$$\Box\,\Phi \equiv \frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) = 0\,. \tag{3.1}$$

Now with the trial solution of the type:

$$\Phi(x^\mu) = \frac{1}{r}\,R(r)\,\exp(i\omega t)\,Y_{lm}(\vartheta, \Phi)\,, \tag{3.2}$$

we can separate the radial equation, which in tortoise coordinate $x = \int \frac{dr}{f(r)}$ is of the form,

$$\frac{d^2R(r)}{dx^2} + \left[\omega^2 - V(r)\right]R(r) = 0\,, \tag{3.3}$$

where $V(r)$ is the Regge-Wheeler potential [28] and is given by,

$$V(r) = f(r)\left[\frac{l(l+1)}{r^2} + \frac{1}{r}\frac{d}{dr}f(r)\right]. \tag{3.4}$$

In our analysis we would use the monodromy method of ref. [7]. We need to extend (3.3) throughout the entire complex r plain in order to use monodromy technique [7].

The boundary condition satisfied by the scalar field $\Phi(x)$ is given by

$$\Phi(x) \sim e^{i\omega x} \text{ as } x \to -\infty, \Phi(x) \sim e^{-i\omega x} \text{ as } x \to +\infty. \tag{3.5}$$

Near the origin $r = 0$, the tortoise coordinate looks like

$$x \sim -\frac{\Gamma(3/2)r^2}{4M\gamma(3/2, r_0^2/4\theta)}. \tag{3.6}$$

Here we have assumed that $r^2/4\theta$ is sufficiently large near origin such that $\frac{\gamma(3/2,r^2/4\theta)}{\Gamma(3/2)}$ can safely be taken outside the integral, when performing the integration for evaluation of x. We can do it by suitably adjusting the non-commutative parameter θ. r_0 is any point near origin, which we have put to keep it outside the integral. The potential $V(r)$ near the origin takes the form,

$$V(r(x)) = \frac{j^2 - 1}{4x^2}, \tag{3.7}$$

where $j = 0$. For highly damped asymptotic quasi-normal modes, we take the frequency ω to be approximately purely imaginary. Thus, for the Stokes line defined by Im $(\omega x) = 0$, x is approximately purely imaginary. This together with Eq. (3.6) implies that near $r = 0$, the behavior of r is of the form

$$r = \rho \, e^{i\pi/4} e^{in\pi/2} \tag{3.8}$$

with $\rho > 0$ and $n = 0, 1, 2$ and 3. The signs of ωx on these lines are given by $(-1)^n$ and near the origin, the Stokes lines are equispaced by an angle $\frac{\pi}{2}$. Also note that near infinity, $x \sim r$ and Re $(x) = 0$ and Re $(r) = 0$ are approximately parallel. Two of the stokes lines are parallel to the two imaginary r axis and other two Stokes lines starting from the origin would form a closed loop in the complex r plane [7].

Now to calculate quasi-normal modes, we will consider the contour of figure (2) drawn in complex r plane in Ref. [7]. The reason why we are considering the figure (2) of Ref. [7] for our calculation needs to be clarified here. In our analysis of quasi-normal modes as we have said earlier that we are considering the case $M \geq M_0$. Although for $M > M_0$, the black hole [26] has two horizons, we can consider it as a single horizon black hole when mass of the black hole M becomes much much larger than the critical mass M_0. So in this situation the non-commutative geometry inspired Schwarzschild black hole spacetime geometry is similar to the Schwarzschild geometry, except the modified horizon. The solution of the wave Eqn. (3.3) near origin is given by

$$\Phi(x) = A_+ \sqrt{2\pi\omega x} J_{\frac{j}{2}}(\omega x) + B_+ \sqrt{2\pi\omega x} J_{-\frac{j}{2}}(\omega x), \tag{3.9}$$

where $J_{\nu=\pm\frac{j}{2}}$ are the Bessel functions of first kind and $A_+,\ B_+$ are constants. Since we are considering the situation where Im $(\omega) \to \infty$, we can use the asymptotic expansion of the Bessel function to write the solution Eq. (3.9) as

$$\Phi(x) = \left(A_+ e^{-i\alpha_+} + B_+ e^{-i\alpha_-}\right) e^{i\omega x} + \left(A_+ e^{i\alpha_+} + B_+ e^{i\alpha_-}\right) e^{-i\omega x}, \tag{3.10}$$

where $\alpha_\pm = \frac{\pi}{4}(1 \pm j)$. Now consider a region near A (A is a point indicated in figure (2) of Ref. [7]), where we have $\omega x \to \infty$ and $x =\to +\infty$. So imposing the boundary condition Eq. (3.5), we get from Eq.(3.10):

$$A_+ e^{-i\alpha_+} + B_+ e^{-i\alpha_-} = 0. \tag{3.11}$$

Again consider the region near B (B is a point indicated in figure (2) of Ref. [7]). Since the stokes lines are equispaced near origin (r=0), there is a difference in angle of $3\pi/2$ in complex r plane, when we pass from region A to region B along the stokes line. In the complex x plane, which amounts to a 3π rotation.

So Bessel function will experience a change of phase $e^{3\pi i}$ while passing from A to B. Taking into account the change in the Bessel function as,

$$\sqrt{2\pi e^{3\pi i}\omega x}\ J_{\pm\frac{j}{2}}\left(e^{3\pi i}\omega x\right) = e^{\frac{3\pi i}{2}(1\pm j)}\sqrt{2\pi\omega x}\ J_{\pm\frac{j}{2}}\left(\omega x\right), \tag{3.12}$$

we can write the solution for $\Phi(x)$ to be of the form,

$$\Phi(x) = \left(A_+ e^{7i\alpha_+} + B_+ e^{7i\alpha_-}\right) e^{i\omega x} + \left(A_+ e^{5i\alpha_+} + B_+ e^{5i\alpha_-}\right) e^{-i\omega x} \tag{3.13}$$

We now close the two asymptotic branches of the Stokes lines by a contour along $r \sim \infty$ on which Re $(x) > 0$. Since we are considering modes with Im $(\omega) \to \infty$, on this part of the contour $e^{i\omega x}$ is exponentially small. So we rely only on the coefficient of $e^{-i\omega x}$ of Eq. (3.13). As the contour is completed, this coefficient picks up a multiplicative factor given by

$$\frac{A_+ e^{5i\alpha_+} + B_+ e^{5i\alpha_-}}{A_+ e^{i\alpha_+} + B_+ e^{i\alpha_-}}. \tag{3.14}$$

The monodromy of $e^{-i\omega x}$ along this clockwise contour is $e^{-\frac{\pi\omega}{k_h}}$ [7], where $k_h = \frac{1}{2}f'(r_h)$ is the surface gravity at the horizon r_h. Thus the complete monodromy of the solution to the wave equation along this clockwise contour is

$$\mathcal{M}(r_h) = \frac{A_+ e^{5i\alpha_+} + B_+ e^{5i\alpha_-}}{A_+ e^{i\alpha_+} + B_+ e^{i\alpha_-}} e^{-\frac{\pi\omega}{k_h}}. \tag{3.15}$$

The contour discussed above can now be smoothly deformed to a small circle going clockwise around the horizon at $r = r_h$. Near $r = r_h$ the potential in the wave equation approximately vanishes. From the boundary condition Eq. (3.5), we see that the solution of the wave equation Eq. (3.3) near the black hole event horizon is of the form

$$\Phi(x) \sim e^{i\omega x}. \tag{3.16}$$

The monodromy of Φ going around the small clockwise circle around the event horizon is thus given by [7]

$$\mathcal{M}(r_h) = e^{\frac{\pi\omega}{k_h}}. \tag{3.17}$$

Since the monodromy around the two contour have to be same, we get from Eq. (3.15) and Eq. (3.17),

$$\frac{A_+ e^{5i\alpha_+} + B_+ e^{5i\alpha_-}}{A_+ e^{i\alpha_+} + B_+ e^{i\alpha_-}} e^{-\frac{\pi\omega}{k_h}} = e^{\frac{\pi\omega}{k_h}}. \tag{3.18}$$

Eliminating the constants A_+ and B_+ from Eq. (3.11) and Eq. (3.18), we get

$$\mathrm{e}^{\frac{2\pi\omega}{k_h}} = -\lim j \to 0 \frac{\sin(\frac{3\pi}{2})j}{\sin(\frac{\pi}{2})j}. \tag{3.19}$$

So from Eq. (3.19), we can immediately write down the analytic expression for asymptotic quasi-normal frequency of the form,

$$\omega = T_h(\theta)\log 3 + 2\pi i T_h(\theta)\left(n + \frac{1}{2}\right), \tag{3.20}$$

where $T_h(\theta) = \frac{k_h}{2\pi}$ is the Hawking temperature of the non-commutative geometry inspired Schwarzschild black hole in $\hbar = 1$ unit.

4 Asymptotic quasi-normal mode analysis for the two horizon black hole

We now calculate asymptotic quasi-normal modes for the case when the non-commutative geometry inspired Schwarzschild black hole has two horizons instead of one as in the above single horizon analysis. The basic calculations are exactly similar to the one in previous section. The only difference comes due to the presence of extra horizon in the scenario. Now the question is, how does this extra horizon modify the results of the previous section? To get an answer to this question we need to look at Eq. (3.15) and Eq. (3.17) where the effect of event horizon comes as the exponential factor. So, for for the extra horizon, now we should get one more exponential factor in both these equations. For the two horizon case now the Eq (3.15) should be replaced by

$$\mathcal{M}(r_h, r_{in}) = \frac{A_+ e^{5i\alpha_+} + B_+ e^{5i\alpha_-}}{A_+ e^{i\alpha_+} + B_+ e^{i\alpha_-}} e^{-\frac{\pi\omega}{k_{in}} - \frac{\pi\omega}{k_h}} , \tag{4.1}$$

and Eq. (3.17) should be replaced by

$$\mathcal{M}(r_h, r_{in}) = e^{\frac{\pi\omega}{k_{in}} + \frac{\pi\omega}{k_h}} . \tag{4.2}$$

This is very standard technique and can be found in the review written by J. Natário and R. Schiappa [9]. Now the monodromy matching between Eq. (4.1) and Eq. (4.2), suggests that

$$\frac{A_+ e^{5i\alpha_+} + B_+ e^{5i\alpha_-}}{A_+ e^{i\alpha_+} + B_+ e^{i\alpha_-}} e^{-\frac{\pi\omega}{k_{in}} - \frac{\pi\omega}{k_h}} = e^{\frac{\pi\omega}{k_{in}} + \frac{\pi\omega}{k_h}} . \tag{4.3}$$

Eliminating the constants A_+ and B_+ from Eq. (3.11) and Eq. (4.3), we get

$$\mathrm{e}^{\frac{2\pi\omega}{k_{in}} + \frac{2\pi\omega}{k_h}} = -\lim j \rightarrow 0 \frac{\sin(\frac{3\pi}{2})j}{\sin(\frac{\pi}{2})j} . \tag{4.4}$$

So from Eq. (4.4), we can immediately write down the analytic expression for asymptotic quasi-normal frequency of the form,

$$\omega = (T_h(\theta) + T_{in}(\theta))\mathrm{log}3 + 2\pi i (T_h(\theta) + T_{in}(\theta)) \left(n + \frac{1}{2}\right) , \tag{4.5}$$

where $T_h(\theta) = \frac{k_h}{2\pi}$ and $T_{in}(\theta) = \frac{k_{in}}{2\pi}$ are the Hawking temperatures of the non-commutative geometry inspired Schwarzschild black hole at r_{in} and r_h respectively. In the $\theta \rightarrow 0$ limit Eq. (4.4) reduce to the Schwarzschild value , because, in this limit $k_{in}(\theta \rightarrow 0) = T_{in}(\theta \rightarrow 0) = \infty$. So we get back the Schwarzschild asymptotic quasinormal modes.

So far we have considered only scalar field perturbation in our calculation. But what happens to the Gravitational and Electromagnetic perturbations? As we know (see appendix of review written by J. Natário and R. Schiappa [9]) Gravitational perturbation can be decoupled into tensor, vector and scalar type perturbations and Electromagnetic perturbation can be decoupled into vector and scalar type perturbations. So we only need to consider tensor, vector and scalar type perturbations. Since in our case we assume the form of $f(r) \sim \frac{1}{r}$ near origin, the value of j in the potential $V(r)$ Eq. (3.4) for different cases are as follows: for scalar $j = 0$, for tensor $j = 0$ and for vector $j = 2$. Taking these values into consideration, we get the same result for the asymptotic quasi-normal frequency for all the three cases.

For the extremal black hole $M = M_0$, we can calculate the asymptotic quasinomal modes exactly similar to the single horizon case done in section 3. But now the surface gravity k_h and Hawking temperature T_h will be replaced by the surface gravity and Hawking temperature at the extremal horizon respectively.

5 Discussion

We have calculated analytically the asymptotic quasi-normal frequencies for the non-commutative geometry inspired Schwarzschild black hole in (3+1) dimensions. Firstly, we have taken the mass of the black hole M to be much much larger than the critical mass M_0 [26]. It enable us to consider the black hole as a single horizon Schwarzschild black hole with correction due to non-commutativity. In this situation, the real part of the asymptotic quasi-normal frequency becomes of the form $\frac{Re(\omega)}{T_h} = \ln(3)$, which is what we get for Schwarzschild black hole. The effect of non-commutativity comes through the Hawking temperature $T_h(\theta)$ and it reduces to the Schwarzschild value when the non-commutativity goes to zero ($\theta = 0$). Then we consider the two horizon case and calculate its asymptotic quasi-normal frequency. We found the real part to be again proportional to $\ln(3)$.

Important point to note is that in our calculation we assumed $r^2/4\theta$ very large near the origin ($r \to 0$) by making θ very small. This constraint can't be relaxed. The reason is the following: In order to remove this constraint, we need to consider the case when θ is large. That means $r^2/4\theta$ is small near the origin ($r \to 0$). For this case the upper incomplete Gamma function should be proportional to $r^{3/2}$, which means $f(r) = 1 - k(\theta)r^2$. $k(\theta)$ is dependent on non-commutative parameter θ. So near origin neglecting the second term in $f(r)$ for the evaluation of tortoise coordinate x, we get $x \sim r$. This means we have now two stokes lines, one along the negative imaginary axis and other along the positive imaginary axis on complex r plane. So now for this case if we want to calculate the asymptotic quasi-normal mode we need to calculate the phase factor of the wave function going from region A to region B (figure 2 of Ref. [7]). This phase factor is π (both r and x have the same phase factor in this case). Taking into account this phase factor if we follow the calculation of section 3, we will get a multiplicative factor 1 instead of Eq. (3.14). This will lead to 1 in the r.h.s of Eq. (3.19), which means asymptotic quasi-normal frequency is proportional to 0!. But this contradicts our assumption that the imaginary part of frequency is very large. We can't resolve it using monodromy technique. Perhaps more careful analysis should be made in order to relax the constraint made on $r^2/4\theta$. Finally generalization of the idea [26] to any dimension ($d > 3$) and study of its quasi-normal modes would be interesting. Study of low damping quasi-normal modes of this kind of black hole is also an open problem. We hope to perform these problems in future.

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